

2014 Jan C34

2. Solve, for $0 \leq x \leq 270^\circ$, the equation

$$\frac{\tan 2x + \tan 50^\circ}{1 - \tan 2x \tan 50^\circ} = 2$$

Give your answers in degrees to 2 decimal places.

(6)

2014 Jan C34

8. (a) Prove that

$$2\operatorname{cosec} 2A - \cot A \equiv \tan A, \quad A \neq \frac{n\pi}{2}, n \in \mathbb{Z} \quad (4)$$

(b) Hence solve, for $0 \leq \theta \leq \frac{\pi}{2}$

(i) $2\operatorname{cosec} 4\theta - \cot 2\theta = \sqrt{3}$

(ii) $\tan \theta + \cot \theta = 5$

Give your answers to 3 significant figures.

(6)

13. (a) Express $2\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$. Give your value of α to 2 decimal places. (3)

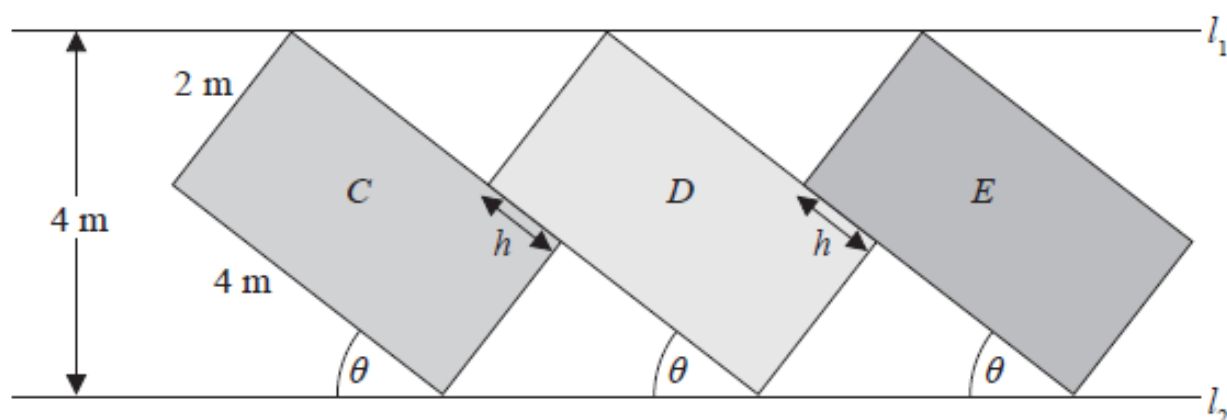


Figure 4

Figure 4 shows the design for a logo that is to be displayed on the side of a large building. The logo consists of three rectangles, C , D and E , each of which is in contact with two horizontal parallel lines l_1 and l_2 . Rectangle D touches rectangles C and E as shown in Figure 4.

Rectangles C , D and E each have length 4 m and width 2 m. The acute angle θ between the line l_2 and the longer edge of each rectangle is shown in Figure 4.

Given that l_1 and l_2 are 4 m apart,

(b) show that

$$2\sin\theta + \cos\theta = 2 \quad (2)$$

Given also that $0 < \theta < 45^\circ$,

(c) solve the equation

$$2\sin\theta + \cos\theta = 2$$

giving the value of θ to 1 decimal place. (3)

Rectangles C and D and rectangles D and E touch for a distance h m as shown in Figure 4.

Using your answer to part (c), or otherwise,

(d) find the value of h , giving your answer to 2 significant figures. (3)

2015 Jan C34

2. Solve, for $0 \leq \theta < 2\pi$,

$$2 \cos 2\theta = 5 - 13 \sin \theta$$

Give your answers in radians to 3 decimal places.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

7. (a) Given that

$$2 \cos(x + 30)^\circ = \sin(x - 30)^\circ$$

without using a calculator, show that

$$\tan x^\circ = 3\sqrt{3} - 4$$

(5)

(b) Hence or otherwise solve, for $0 \leq \theta < 180$,

$$2 \cos(2\theta + 40)^\circ = \sin(2\theta - 20)^\circ$$

Give your answers to one decimal place.

(4)

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$

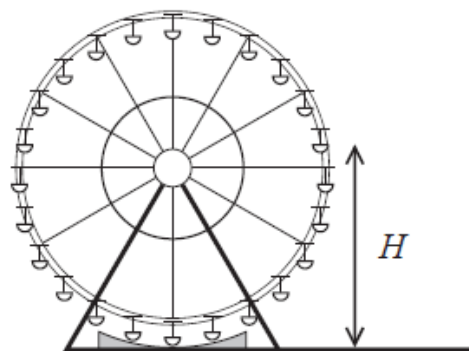
Give the exact value of R and give the value of α to 2 decimal places.

(3)

Alana models the height above the ground of a passenger on a Ferris wheel by the equation

$$H = 12 - 10\cos(30t)^\circ + 3\sin(30t)^\circ$$

where the height of the passenger above the ground is H metres at time t minutes after the wheel starts turning.



- (b) Calculate

- (i) the maximum value of H predicted by this model,
- (ii) the value of t when this maximum first occurs.

Give each answer to 2 decimal places.

(4)

- (c) Calculate the value of t when the passenger is 18m above the ground for the first time.

Give your answer to 2 decimal places.

(4)

- (d) Determine the time taken for the Ferris wheel to complete two revolutions.

(2)

2015 June C34

7. (a) Use the substitution $t = \tan x$ to show that the equation

$$4 \tan 2x - 3 \cot x \sec^2 x = 0$$

can be written in the form

$$3t^4 + 8t^2 - 3 = 0$$

(4)

- (b) Hence solve, for $0 \leq x < 2\pi$,

$$4 \tan 2x - 3 \cot x \sec^2 x = 0$$

Give each answer in terms of π . You must make your method clear.

(4)

2015 June C34

11. (a) Express $1.5 \sin \theta - 1.2 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of R and the value of α to 3 decimal places.

(3)

The height, H metres, of sea water at the entrance to a harbour on a particular day, is modelled by the equation

$$H = 3 + 1.5 \sin\left(\frac{\pi t}{6}\right) - 1.2 \cos\left(\frac{\pi t}{6}\right), \quad 0 \leq t < 12$$

where t is the number of hours after midday.

- (b) Using your answer to part (a), calculate the minimum value of H predicted by this model and the value of t , to 2 decimal places, when this minimum occurs.

(4)

- (c) Find, to the nearest minute, the times when the height of sea water at the entrance to the harbour is predicted by this model to be 4 metres.

(6)

2016 Jan C34

2. (a) Show that

$$\cot^2 x - \operatorname{cosec} x - 11 = 0$$

may be expressed in the form $\operatorname{cosec}^2 x - \operatorname{cosec} x + k = 0$, where k is a constant.

(1)

- (b) Hence solve for $0 \leq x < 360^\circ$

$$\cot^2 x - \operatorname{cosec} x - 11 = 0$$

Give each solution in degrees to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

2016 Jan C34

8.

$$f(\theta) = 9\cos^2\theta + \sin^2\theta$$

- (a) Show that $f(\theta) = a + b\cos 2\theta$, where a and b are integers which should be found.

(3)

10. (a) Express $3 \sin 2x + 5 \cos 2x$ in the form $R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α to 3 significant figures.

(3)

- (b) Solve, for $0 < x < \pi$,

$$3 \sin 2x + 5 \cos 2x = 4$$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

$$g(x) = 4(3 \sin 2x + 5 \cos 2x)^2 + 3$$

- (c) Using your answer to part (a) and showing your working,

(i) find the greatest value of $g(x)$,

(ii) find the least value of $g(x)$.

(4)

2016 June C34

1. (a) Express $3 \cos \theta + 5 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

- (b) Hence solve, for $0 \leq \theta < 360^\circ$, the equation

$$3 \cos \theta + 5 \sin \theta = 2$$

Give your answers to one decimal place.

(4)

- (c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$3 \cos \theta - 5 \sin \theta = 2$$

(2)

2016 June C34

8. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \quad x \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z} \quad (4)$$

(b) Hence solve, for $0 \leq \theta < \frac{\pi}{2}$

(i) $\sin 2\theta - \tan \theta = \sqrt{3} \cos 2\theta$

(ii) $\tan(\theta + 1) \cos(2\theta + 2) - \sin(2\theta + 2) = 2$

Give your answers in radians to 3 significant figures, as appropriate.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(7)

2017 Jan C34

8. (a) Using the trigonometric identity for $\tan(A + B)$, prove that

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}, \quad x \neq (2n + 1)30^\circ, \quad n \in \mathbb{Z} \quad (4)$$

- (b) Hence solve, for $-30^\circ < x < 30^\circ$,

$$\tan 3x = 11 \tan x$$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

2017 Jan C34

11. (a) Express $35 \sin x - 12 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R , and give the value of α , in radians, to 4 significant figures. (3)

- (b) Hence solve, for $0 \leq x < 2\pi$,

$$70 \sin x - 24 \cos x = 37$$

(Solutions based entirely on graphical or numerical methods are not acceptable.) (4)

$$y = \frac{7000}{31 + (35 \sin x - 12 \cos x)^2}, \quad x > 0$$

- (c) Use your answer to part (a) to calculate

- (i) the minimum value of y ,
- (ii) the smallest value of x , $x > 0$, at which this minimum value occurs.

(4)

2017 June C34

7. (a) Prove that

$$\frac{1 - \cos 2x}{1 + \cos 2x} \equiv \tan^2 x, \quad x \neq (2n + 1)90^\circ, n \in \mathbb{Z} \quad (3)$$

(b) Hence, or otherwise, solve, for $-90^\circ < \theta < 90^\circ$,

$$\frac{2 - 2 \cos 2\theta}{1 + \cos 2\theta} - 2 = 7 \sec \theta$$

Give your answers in degrees to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

2017 June C34

10. (a) Write $2 \sin \theta - \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha \leq 90^\circ$. Give the exact value of R and give the value of α to one decimal place. (3)

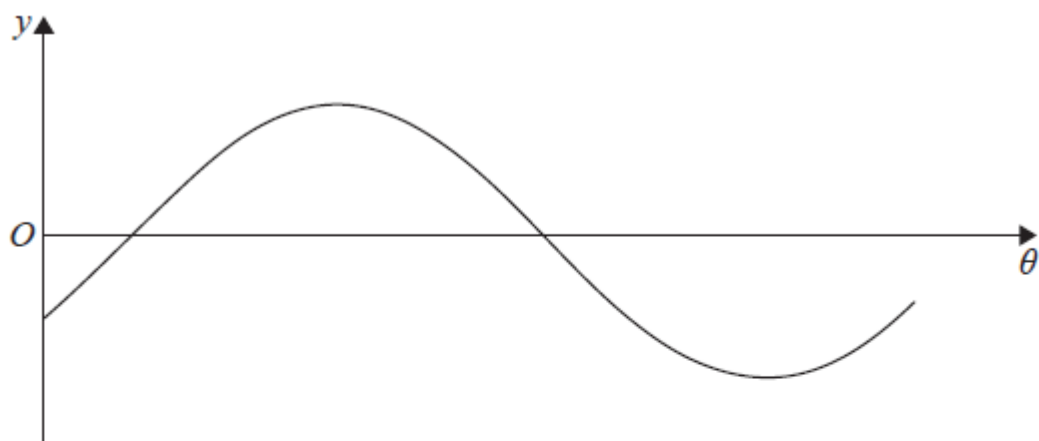


Figure 3

Figure 3 shows a sketch of the graph with equation $y = 2 \sin \theta - \cos \theta$, $0 \leq \theta < 360^\circ$

- (b) Sketch the graph with equation

$$y = |2 \sin \theta - \cos \theta|, \quad 0 \leq \theta < 360^\circ$$

stating the coordinates of all points at which the graph meets or cuts the coordinate axes.

1

(3)

The temperature of a warehouse is modelled by the equation

$$f(t) = 5 + |2 \sin(15t)^\circ - \cos(15t)^\circ|, \quad 0 \leq t < 24$$

where $f(t)$ is the temperature of the warehouse in degrees Celsius and t is the time measured in hours from midnight.

State

- (c) (i) the maximum value of $f(t)$,
(ii) the largest value of t , for $0 \leq t < 24$, at which this maximum value occurs. Give your answer to one decimal place.

(3)

2017 Oct C34

4. (a) Prove that

$$\frac{1 - \cos 2x}{\sin 2x} \equiv \tan x, \quad x \neq \frac{n\pi}{2} \quad (3)$$

- (b) Hence solve, for $0 \leq \theta < 2\pi$,

$$3\sec^2 \theta - 7 = \frac{1 - \cos 2\theta}{\sin 2\theta}$$

Give your answers in radians to 3 decimal places, as appropriate.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

9. (a) Show that

$$\frac{\cot^2 x}{1 + \cot^2 x} \equiv \cos^2 x \quad (3)$$

(b) Hence solve, for $0 \leq x < 360^\circ$,

$$\frac{\cot^2 x}{1 + \cot^2 x} = 8 \cos 2x + 2 \cos x$$

Give each solution in degrees to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

12. (a) Express $2\sin x - 4\cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α , in radians, to 3 significant figures.
(3)

In a town in Norway, a student records the number of hours of daylight every day for a year. He models the number of hours of daylight, H , by the continuous function given by the formula

$$H = 12 + 4\sin\left(\frac{2\pi t}{365}\right) - 8\cos\left(\frac{2\pi t}{365}\right), \quad 0 \leq t \leq 365$$

where t is the number of days since he began recording.

- (b) Using your answer to part (a), or otherwise, find the maximum and minimum number of hours of daylight given by this formula. Give your answers to 3 significant figures.
(3)
- (c) Use the formula to find the values of t when $H = 17$, giving your answers to the nearest integer.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

2018 June C34

6. (a) Express $\sqrt{5} \cos \theta - 2 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

State the value of R and give the value of α to 4 significant figures.

(3)

- (b) Solve, for $-\pi < \theta < \pi$,

$$\sqrt{5} \cos \theta - 2 \sin \theta = 0.5$$

giving your answers to 3 significant figures.

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(4)

$$f(x) = A(\sqrt{5} \cos \theta - 2 \sin \theta) + B \quad \theta \in \mathbb{R}$$

where A and B are constants.

Given that the range of f is

$$-15 \leq f(x) \leq 33$$

- (c) find the value of B and the possible values of A .

(4)

2018 June C34

12. (a) Show that

$$\cot x - \tan x \equiv 2 \cot 2x, \quad x \neq 90n^\circ, n \in \mathbb{Z} \quad (4)$$

(b) Hence, or otherwise, solve, for $0 \leq \theta < 180^\circ$

$$5 + \cot(\theta - 15^\circ) - \tan(\theta - 15^\circ) = 0$$

giving your answers to one decimal place.

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(5)

2018 Oct C34

1. (a) Write $\cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

- (b) Hence solve, for $0 \leq \theta < \pi$, the equation

$$\cos 2\theta + 4 \sin 2\theta = 1.2$$

giving your answers to 2 decimal places.

(5)

2018 Oct C34

3. Given

$$\cos \theta^\circ = p, \text{ where } p \text{ is a constant and } \theta^\circ \text{ is acute}$$

use standard trigonometric identities to find, in terms of p ,

(a) $\sec \theta^\circ$ (1)

(b) $\sin(\theta - 90)^\circ$ (2)

(c) $\sin 2\theta^\circ$ (3)

Write each answer in its simplest form.

2019 Jan C34

1. (a) Express $7 \sin 2\theta - 2 \cos 2\theta$ in the form $R \sin(2\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

- (b) Hence solve, for $0 \leq \theta < 90^\circ$, the equation

$$7 \sin 2\theta - 2 \cos 2\theta = 4$$

giving your answers in degrees to one decimal place.

(4)

- (c) Express $28 \sin \theta \cos \theta + 8 \sin^2 \theta$ in the form $a \sin 2\theta + b \cos 2\theta + c$, where a , b and c are constants to be found.

(3)

- (d) Use your answers to part (a) and part (c) to deduce the exact maximum value of $28 \sin \theta \cos \theta + 8 \sin^2 \theta$

(2)

2019 Jan C34

5. The angle x and the angle y are such that

$$\tan x = m \quad \text{and} \quad 4 \tan y = 8m + 5$$

where m is a constant.

Given that $16 \sec^2 x + 16 \sec^2 y = 537$

- (a) find the two possible values of m .

(4)

Given that the angle x and the angle y are acute, find the exact value of

- (b) $\sin x$

(2)

- (c) $\cot y$

(2)

2019 June C34

7. (a) Express $5 \cos \theta - 3 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α , in radians, to 4 decimal places. (3)

The height of sea water, H metres, on a harbour wall is modelled by the equation

$$H = 6 + 2.5 \cos\left(\frac{4\pi t}{25}\right) - 1.5 \sin\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12$$

where t is the number of hours after midday.

- (b) Calculate the times at which the model predicts that the height of sea water on the harbour wall will be 4.6 metres. Give your answers to the nearest minute. (6)

2019 June C34

9. (a) Using the formula for $\sin(A + B)$ and the relevant double angle formulae, find an identity for $\sin 3x$, giving your answer in the form

$$\sin(3x) \equiv P \sin x + Q \sin^3 x$$

where P and Q are constants to be determined.

(4)

2020 Jan P3

5. (a) Use the substitution $t = \tan x$ to show that the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

can be written in the form

$$5t^4 - 24t^2 - 5 = 0 \quad (4)$$

- (b) Hence solve, for $0 \leq x < 360^\circ$, the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

Show each stage of your working and give your answers to one decimal place.

(4)

9.

$$f(\theta) = 5 \cos \theta - 4 \sin \theta \quad \theta \in \mathbb{R}$$

- (a) Express $f(\theta)$ in the form $R \cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

The curve with equation $y = \cos \theta$ is transformed onto the curve with equation $y = f(\theta)$ by a sequence of two transformations.

Given that the first transformation is a stretch and the second a translation,

- (b) (i) describe fully the transformation that is a stretch,
(ii) describe fully the transformation that is a translation.

(2)

Given

$$g(\theta) = \frac{90}{4 + (f(\theta))^2} \quad \theta \in \mathbb{R}$$

- (c) find the range of g .

(2)

2020 Oct P3

1. Solve, for $0 \leq x < 360^\circ$, the equation

$$2\cos 2x = 7\cos x$$

giving your solutions to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

2020 Oct P3

5. (a) Show that

$$\sin 3x \equiv 3 \sin x - 4 \sin^3 x$$

(4)

2020 Oct P3

7. (a) Express $\cos x + 4 \sin x$ in the form $R \cos(x - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

A scientist is studying the behaviour of seabirds in a colony.

She models the height above sea level, H metres, of one of the birds in the colony by the equation

$$H = \frac{24}{3 + \cos\left(\frac{1}{2}t\right) + 4\sin\left(\frac{1}{2}t\right)} \quad 0 \leq t \leq 6.5$$

where t seconds is the time after it leaves the nest.

Find, according to the model,

- (b) the minimum height of the seabird above sea level, giving your answer to the nearest cm,

(2)

- (c) the value of t , to 2 decimal places, when $H = 10$

(4)

2021 Jan P3

7. (a) Prove that

$$\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} = \operatorname{cosec} x \quad x = \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$7 + \frac{\sin 4\theta}{\cos 2\theta} + \frac{\cos 4\theta}{\sin 2\theta} = 3\cot^2 2\theta$$

giving your answers in radians to 3 significant figures where appropriate.

(6)

2021 June P3

2. (a) Show that

$$\frac{1 - \cos 2x}{2 \sin 2x} \equiv k \tan x \quad x \neq (90n)^\circ \quad n \in \mathbb{Z}$$

where k is a constant to be found.

(3)

(b) Hence solve, for $0 < \theta < 90^\circ$

$$\frac{9(1 - \cos 2\theta)}{2 \sin 2\theta} = 2 \sec^2 \theta$$

giving your answers to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

2021 June P3

9. (a) Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians, to 3 decimal places.

(3)

The function g is defined by

$$g(\theta) = 10 + 12\sin\left(2\theta - \frac{\pi}{6}\right) - 5\cos\left(2\theta - \frac{\pi}{6}\right) \quad \theta > 0$$

Find

- (b) (i) the minimum value of $g(\theta)$
(ii) the smallest value of θ at which the minimum value occurs.

(3)

The function h is defined by

$$h(\beta) = 10 - (12\sin\beta - 5\cos\beta)^2$$

- (c) Find the range of h .

(2)

2021 Oct P3

4. In this question you should show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2 \sin (\theta - 30^\circ) = 5 \cos \theta$$

can be written in the form

$$\tan \theta = 2 \sqrt{3}$$

(4)

(b) Hence, or otherwise, solve for $0 \leq x \leq 360^\circ$

$$2 \sin (x - 10^\circ) = 5 \cos (x + 20^\circ)$$

giving your answers to one decimal place.

(3)

2021 Oct P3

8. A curve C has equation $y = f(x)$, where

$$f(x) = \arcsin\left(\frac{1}{2}x\right) \quad -2 \leq x \leq 2 \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

(a) Sketch C .

(1)

10.

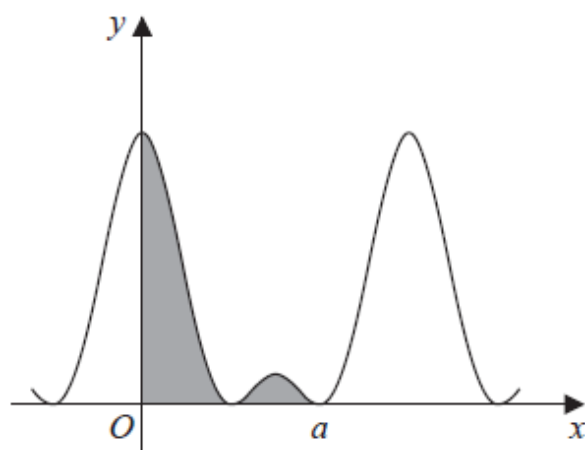


Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = (1 + 2 \cos 2x)^2$$

(a) Show that

$$(1 + 2 \cos 2x)^2 \equiv p + q \cos 2x + r \cos 4x$$

where p , q and r are constants to be found.

(2)

2022 Jan P3

2. (a) Show that the equation

$$8 \cos \theta = 3 \operatorname{cosec} \theta$$

can be written in the form

$$\sin 2\theta = k$$

where k is a constant to be found.

(3)

- (b) Hence find the smallest positive solution of the equation

$$8 \cos \theta = 3 \operatorname{cosec} \theta$$

giving your answer, in degrees, to one decimal place.

(2)

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

- (i) Solve, for $0 < x \leq \pi$, the equation

$$2 \sec^2 x - 3 \tan x = 2$$

giving the answers, as appropriate, to 3 significant figures.

(4)

- (ii) Prove that

$$\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \equiv 2$$

(4)

2022 June P3

7. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2 \sin \theta (3 \cot^2 2\theta - 7) = 13 \sec \theta$$

can be written as

$$3 \operatorname{cosec}^2 2\theta - 13 \operatorname{cosec} 2\theta - 10 = 0$$

(4)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$, the equation

$$2 \sin \theta (3 \cot^2 2\theta - 7) = 13 \sec \theta$$

giving your answers to 3 significant figures.

(4)

9.

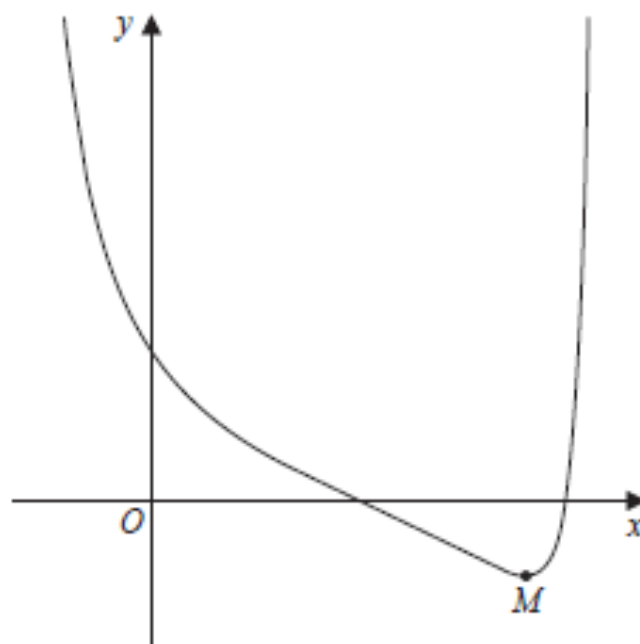


Figure 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 5 shows the curve with equation

$$y = \frac{1 + 2 \cos x}{1 + \sin x} \quad -\frac{\pi}{2} < x < \frac{3\pi}{2}$$

The point M , shown in Figure 5, is the minimum point on the curve.

(a) Show that the x coordinate of M is a solution of the equation

$$2 \sin x + \cos x = -2 \quad (4)$$

(b) Hence find, to 3 significant figures, the x coordinate of M . (5)

6.

$$y = \frac{2 + 3\sin x}{\cos x + \sin x}$$

Show that

$$\frac{dy}{dx} = \frac{a \tan x + b \sec x + c}{\sec x + 2 \sin x}$$

where a , b and c are integers to be found.

(6)

8. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

- (a) Express $8 \sin x - 15 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R , and give the value of α , in radians, to 4 significant figures. (3)

$$f(x) = \frac{15}{41 + 16 \sin x - 30 \cos x} \quad x > 0$$

- (b) Find

- (i) the minimum value of $f(x)$
 (ii) the smallest value of x at which this minimum value occurs. (4)

- (c) State the y coordinate of the minimum points on the curve with equation

$$y = 2f(x) - 5 \quad x > 0 \quad (1)$$

- (d) State the smallest value of x at which a maximum point occurs for the curve with equation

$$y = -f(2x) \quad x > 0 \quad (1)$$

2023 Jan P3

2.

$$f(x) = \cos x + 2 \sin x$$

(a) Express $f(x)$ in the form $R \cos(x - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

$$g(x) = 3 - 7f(2x)$$

(b) Using the answer to part (a),

(i) write down the exact maximum value of $g(x)$,

(ii) find the smallest positive value of x for which this maximum value occurs, giving your answer to 2 decimal places.

(3)

2023 Jan P3

5. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4 \cot 2x \operatorname{cosec} 2x \quad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (4)$$

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4 \cot 2\theta \operatorname{cosec} 2\theta = 2 \tan^2 \theta$$

giving your answers to 2 decimal places.

(5)

2023 June P3

5. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$

$$(x - 2)(\sqrt{3} \sec x + 2) = 0 \quad (3)$$

(ii) Solve, for $0 < \theta < 360^\circ$

$$10 \sin \theta = 3 \cos 2\theta \quad (4)$$

2023 June P3

9.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \quad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta} \right)^2 = 6 \cot \theta - 4$$

giving your answers to 3 significant figures as appropriate.

(5)

2023 Oct P3

3. (a) Using the identity for $\cos(A + B)$, prove that

$$\cos 2A \equiv 2 \cos^2 A - 1$$

(2)

2023 Oct P3

8. (a) Prove that

$$2 \operatorname{cosec}^2 2\theta (1 - \cos 2\theta) \equiv 1 + \tan^2 \theta \quad (4)$$

(b) Hence solve for $0 < x < 360^\circ$, where $x \neq (90n)^\circ$, $n \in \mathbb{N}$, the equation

$$2 \operatorname{cosec}^2 2x (1 - \cos 2x) = 4 + 3 \sec x$$

giving your answers to one decimal place.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

9. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$\frac{3 \sin \theta \cos \theta}{\cos \theta + \sin \theta} = (2 + \sec 2\theta)(\cos \theta - \sin \theta)$$

can be written in the form

$$3 \sin 2\theta - 4 \cos 2\theta = 2 \quad (3)$$

(b) Hence solve for $\pi < x < \frac{3\pi}{2}$

$$\frac{3 \sin x \cos x}{\cos x + \sin x} = (2 + \sec 2x)(\cos x - \sin x)$$

giving the answer to 3 significant figures.

(5)